Stability of Steady Sideslip Equilibria for High Alpha

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The stability of an aircraft with a swept-forward wing at high angles of attack and sideslip is determined analytically via two methods. The first of these considers an eigenvalue analysis of high angle of attack and sideslip trim conditions by which an α-β stability envelope is determined. These results are compared with an approximate analysis which uncouples the translation and rotational motion of the aircraft and analyzes the stability of the latter. This method requires the knowledge of only static derivatives. This second approach reduces to the familiar $C_{n_{\beta_{dyn}}}$ criteria for symmetric flight. The results of the two methods are shown to yield good agreement.

Nomenclature

b	= wing span
$\bar{c}$	= wing chord length
C_T, C_W	= dimensionless thrust and weight coefficients, $C_T = T/qS$, $C_W = W/qS$
C_x, C_y, C_z	= dimensionless body axes force coefficients
C_l, C_m, C_n	= dimensionless rolling, pitching, and yawing moment coefficients, $C_l = L/qSb$, $C_n = N/qSb$, $C_m = M/qSc$
$C_{n_{\beta_{dyn}}}$	= lateral stability parameter, $C_{n_{\beta_{dyn}}} = C_{n_\beta}\cos\alpha - (I_{zz}/I_{xx})C_{l_\beta}\sin\alpha$
I_{xx}, I_{yy}, I_{zz}	= principal moments of inertia about body axes x,y,z
L,M,N	= rolling, pitching, and yawing moments
p,q,r	= roll, pitch, and yaw rates
q	= dynamic pressure, $q = \frac{1}{2}\rho V^2$
S	= wing area
V	= aircraft velocity
α	= angle of attack
β	= sideslip angle
$\delta_r, \delta_a, \delta_e$	= rudder, aileron, and elevator deflections
θ, ϕ, ψ	= pitch, roll, and yaw angles
ρ	= air density

Introduction

TECHNOLOGICAL developments in the realm of supersonic, highly maneuverable fighter aircraft have prompted research into hybrid designs. One such design is the forward-swept wing fighter aircraft. Advantages such as weight reduction, optimum integration of aerodynamic and structural components, increased negative static margin for increased maneuverability, and extended performance envelopes make this design an attractive candidate for future aircraft.

The stability of an aircraft about high angle of attack and high sideslip equilibrium conditions serves as an indication of the aircraft's motion in the neighborhood of these equilibria. Plotting the boundary in an alpha-beta parameter plane of stable equilibrium points yields an indication of the aircraft's allowable alpha-beta combinations for avoiding out-of-control motions. The determination of such boundaries is the subject of Ref. 1. The method of Ref. 1 requires determining the equilibrium points and solving the resulting eigenvalue problem in order to determine the stability of each point in the alpha-beta plane. This is a tedious procedure and various authors have used approximate methods to define the stability boundary. The familiar parameter $C_{n_{\beta_{dyn}}}$ first defined by Moul and Paulson[2] has been used to determine stability bounds for both high angle of attack with zero sideslip and for high angles of attack with sideslip.[3] The parameter is easy to use but is not a reliable predictor in many cases of interest.

Recently Kalviste[4] suggested an improved static analysis technique which couples the lateral and longitudinal equations and yields much more reliable results. The technique is easy to use as it involves evaluating only algebraic relationships.

The current study compares the results of an eigenvalue analysis and the technique of Ref. 4 for a fighter aircraft with a swept-forward wing.

Method of Analysis

Eigenvalue Analysis

In order to determine the stability of an arbitrary aircraft equilibrium position it is necessary to first solve the nonlinear algebraic equations defining the equilibrium. The equations of motion are then linearized about the equilibrium position and an eigenvalue analysis is performed on the resulting linear equations. Our interest will be in the steady sideslip equilibrium condition in which the aircraft angular rates are zero but the sideslip angle (beta) is nonzero.

Steady Sideslip Equilibrium Condition

In order to determine equilibrium conditions we note that in general we require that

$$\dot{V}_e = \dot{\alpha}_e = \dot{\beta}_e = \dot{p}_e = \dot{q}_e = \dot{r}_e = \dot{\theta}_e = \dot{\phi}_e = 0$$

For the case of a straight sideslip we also require that

$$p_e = q_e = r_e = 0$$

Applying these conditions to the equations of motion[6,7] yields

$$C_T + C_x - C_w\sin\theta = 0 \tag{1}$$

$$C_y + C_w\cos\theta\sin\phi = 0 \tag{2}$$

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$$C_z + C_w \cos\phi \cos\theta = 0 \qquad (3)$$

$$C_m + C_{m_{\delta_e}} \delta_e = 0 \qquad (4)$$

$$C_l + C_{l_{\delta_a}} \delta_a + C_{l_{\delta_r}} \delta_r = 0 \qquad (5)$$

$$C_n + C_{n_{\delta_a}} \delta_a + C_{n_{\delta_r}} \delta_r = 0 \qquad (6)$$

The solution of these six algebraic equations represents an equilibrium condition. The six equations contain eleven unknowns and hence an infinite number of solutions exist. For the present analysis the equilibrium values of α, β, ρ, θ, and W will be chosen arbitrarily and Eqs. (1-6) will be solved to determine δ_{e_e}, δ_{a_e}, δ_{r_e}, ϕ_e, V_e, and C_{T_e}. Specifically, from Eqs. (4-6),

$$\delta_{e_e} = -C_{m_e} / C_{m_{\delta_e}} |_e \qquad (7)$$

$$\delta_{r_e} = (C_l C_{n_{\delta_a}} - C_n C_{l_{\delta_a}}) |_e / (C_{n_{\delta_r}} C_{l_{\delta_a}} - C_{n_{\delta_a}} C_{l_{\delta_r}}) |_e \qquad (8)$$

$$\delta_{a_e} = -(C_l + C_{l_{\delta_r}} \delta_r) |_e / C_{l_{\delta_a}} |_e \qquad (9)$$

Since the control derivatives $C_{m_{\delta_e}}$, $C_{n_{\delta_a}}$, $C_{l_{\delta_a}}$, $C_{n_{\delta_r}}$, and $C_{l_{\delta_r}}$ are functions of the control settings, Eqs. (7-9) must be solved iteratively. Once the equilibrium control angles δ_{e_e}, δ_{a_e}, and δ_{r_e} have been determined, we obtain

$$\phi_e = \tan^{-1} [C_{y_e} / C_{z_e}] \qquad (10)$$

$$C_{T_e} = -C_{x_e} + C_w \sin\theta_e \qquad (11)$$

and

$$V_e = [2w / \rho_e S C_{w_e}]^{1/2} \qquad (12)$$

where

$$C_{w_e} = -C_{z_e} / \cos\phi_e \cos\theta_e \qquad (13)$$

The force coefficients C_x, C_y, and C_z are in general functions of α and β as well as the control deflections δ_e, δ_r, and δ_a. Using the equilibrium solution a stability analysis may now be performed.

Stability Analysis/Eigenvalue Analysis

Due to the inherent difficulty in obtaining a closed-form solution to the equations of motion, the equations are linearized about an equilibrium condition and investigated for stability. The characteristic roots or eigenvalues of these equations define the stability of the system at the specified equilibrium condition. This procedure yields the equations, when written in matrix format, to be of the form

$$[A]\{\dot{\hat{X}}\} = [B]\{\hat{X}\} + [C]\{\hat{U}\} \qquad (14)$$

where

$$\{\hat{X}\} = [\Delta V, \Delta\alpha, \Delta q, \Delta\Theta, \Delta\beta, \Delta r, \Delta p, \Delta\phi]^T$$

and

$$\{\hat{U}\} = [\delta_e, \delta_T, \delta_a, \delta_r]$$

The elements of the $[A]$, $[B]$, and $[C]$ matrices are determined using small-perturbation theory and depend upon the equilibrium condition. In order to determine the stability of Eq. (14) we require only the matrices $[A]$ and $[B]$. Specifically Eq. (14) may be rewritten in the form

$$\{\dot{\hat{X}}\} = [S]\{\hat{X}\} \qquad (15)$$

where

$$[S] = [A]^{-1}[B] \qquad (16)$$

The stability of Eq. (15) and hence the system requires that the eigenvalues of $[S]$ all have negative real parts.[5] The stability analysis based on eigenvalues, while conceptually simple, is quite tedious. Specifically, in order to determine the stability of each equilibrium point it is necessary to solve for the eigenvalues of an 8×8 matrix. Since we wish to find the stability boundaries, this must be done numerous times. The resulting computational requirement is therefore sizeable.

Approximate Stability Analysis

An approach developed by Kalviste,[4] although less stringent in application, yields alpha-beta boundaries with far less effort using only static stability derivatives.

This method partitions the aircraft's six degree-of-freedom equations of motion into rotational and translational equations of motion without neglecting the cross-coupling effects which occur between the longitudinal and lateral-directional modes of motion.

The stability of the aircraft is shown in Ref. 4 to be given by the stability of the following coupled equations:

$$\ddot{\alpha} = M_{\alpha_{\text{dyn}}} \Delta\alpha + M_{\beta_{\text{dyn}}} \Delta\beta \qquad (17)$$

$$\ddot{\beta} = -N_{\alpha_{\text{dyn}}} \Delta\alpha - N_{\beta_{\text{dyn}}} \Delta\beta \qquad (18)$$

where

$$M_{\alpha_{\text{dyn}}} = \frac{\partial M_{\text{dyn}}}{\partial \alpha}\bigg|_e \qquad M_{\beta_{\text{dyn}}} = \frac{\partial M_{\text{dyn}}}{\partial \beta}\bigg|_e$$

$$N_{\alpha_{\text{dyn}}} = \frac{\partial N_{\text{dyn}}}{\partial \alpha}\bigg|_e \qquad N_{\beta_{\text{dyn}}} = \frac{\partial N_{\text{dyn}}}{\partial \beta}\bigg|_e$$

and

$$N_{\text{dyn}} = (N/I_{zz})\cos\alpha - (L/I_{xx})\sin\alpha \qquad (19)$$

$$M_{\text{dyn}} = (M/I_{yy}) - [(L/I_{xx})\cos\alpha + (N/I_{zz})\sin\alpha]\tan\beta \qquad (20)$$

The solution to Eqs. (17) and (18) are stable if the roots of the characteristic equation

$$\lambda^2 + \lambda(N_{\beta_{\text{dyn}}} - M_{\alpha_{\text{dyn}}}) + (N_{\alpha_{\text{dyn}}} M_{\beta_{\text{dyn}}} - N_{\beta_{\text{dyn}}} M_{\alpha_{\text{dyn}}}) = 0 \qquad (21)$$

are purely negative. This requires that

$$X = N_{\beta_{\text{dyn}}} - M_{\alpha_{\text{dyn}}} > 0 \qquad (22)$$

$$Y = N_{\alpha_{\text{dyn}}} M_{\beta_{\text{dyn}}} - N_{\beta_{\text{dyn}}} M_{\alpha_{\text{dyn}}} > 0 \qquad (23)$$

and

$$X^2 > 4Y \qquad (24)$$

It is interesting to note that when $N_{\alpha_{\text{dyn}}}$ and $M_{\beta_{\text{dyn}}}$ are zero, the system is stable if $N_{\beta_{\text{dyn}}} > 0$ and $M_{\alpha_{\text{dyn}}} < 0$. The first of these conditions is the familiar $C_{n_{\beta_{\text{dyn}}}}$ criterion in dimensional form.

The algebraic criteria of Eqs. (22-24) can be evaluated for a given equilibrium point. By evaluating their criteria throughout the α-β plane, bounds on stability may be determined.

Results

Steady Sideslip Equilibrium Stability Analysis

Equilibrium conditions were calculated for an alpha range of 0-5 units and a beta range of 0-4 units. The computer program used determined trimmed values of the state

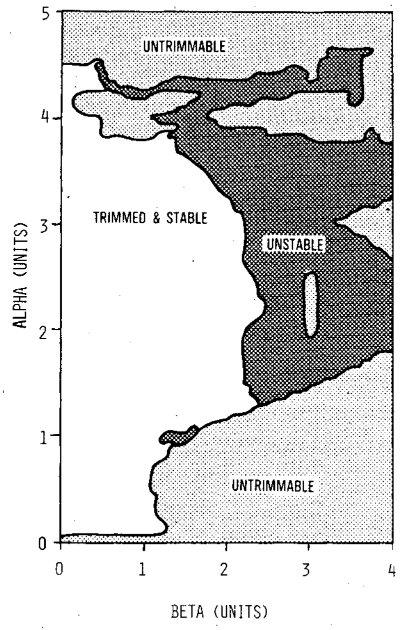

Fig. 1 Stability boundaries, eigenvalue analysis.

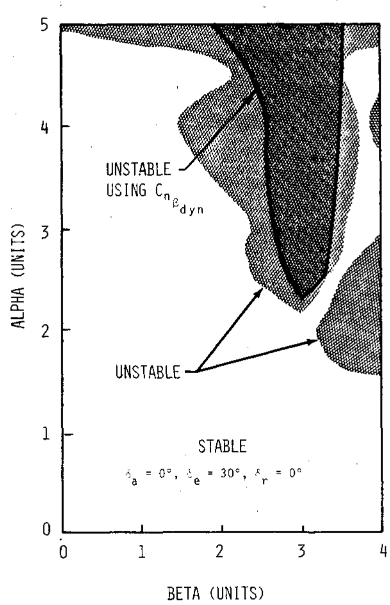

Fig. 2 Stability boundaries, approximate solutions.

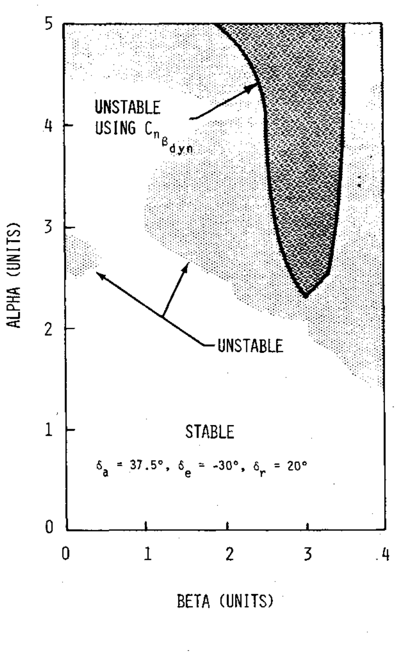

Fig. 3 Stability boundaries, approximate solutions.

variables and analyzed the corresponding state equations for stability using eigenvalue analysis.

As a result of this analysis, alpha-beta boundaries were prepared. Figure 1 gives these boundaries for the prototype investigated with areas of instability and areas where an equilibrium trimmed condition was unobtainable designated.

Approximate Stability Analysis

The alpha-beta boundaries were also calculated using the approximate method developed by Kalviste.[4] As previously mentioned, this technique requires the evaluation of three algebraic criteria which involve only the static aerodynamic derivatives. The system is unstable if any of the three inequalities is violated. This analysis was carried out and Figs. 2 and 3 portray the resulting alpha-beta boundaries for two different combinations of control settings.

The alpha-beta boundaries were also determined for the swept-forward wing (SFW) aircraft using the classical $Cn_{\beta_{\rm dyn}}$ parameter. The results of using the parameter are shown on Figs. 2 and 3 by the dotted lines. Specifically, these lines denote the locus of points for which $C_{n_{\beta_{\rm dyn}}}=0$.

Correlation of Results

Alpha-Beta Boundaries

The alpha-beta boundaries for the SFW derived by use of the two different techniques were compared for the ranges of $0 \le \alpha \le 5$ and $0 \le \beta \le 4$. Stability trends were identified by the eigenvalue analysis of the sideslip equilibrium condition, which the static stability derivative analysis failed to predict accurately. Further, the identification of areas in which the equilibrium-trimmed flight condition was unobtainable restricted the direct correlation between the resultant alpha-beta boundaries. Nevertheless, comparative analysis led to some specific observations:

1) At the lower ranges of alpha and beta, $\alpha < 2.5$ units, $\beta \le 2$ units, independent of control surface deflection, direct comparison yields nearly identical results.

2) For similar ranges of β as in observation 1, but with high angles of attack, similar results are obtained. Further, the approximate analysis results indicate unstable regions which were predicted by the exact solutions to be untrimmable.

Classical $Cn_{\beta_{\rm dyn}}$ and Stability Characteristics

Investigation of Figs. 2 and 3 reveals the difference in predicted stability trends as obtained by the classical $Cn_{\beta_{\rm dyn}}$ analysis and the more stringent analysis using the coupled static stability derivatives. Since the aerodynamic coefficients for the control surfaces were functionally related to only alpha, the $Cn_{\beta_{\rm dyn}}$ curve is therefore independent of any control surface deflections and remains constant.

Correlation of $C_{n_{\beta_{\rm dyn}}}$ and Kalviste's parameters yielded the following results:

1) For all control surface deflections, the stability characteristics method yielded larger areas of instability than predicted by $Cn_{\beta_{\rm dyn}}$ alone.

2) Results obtained tended to show areas of instability at the higher ranges of alpha, $\alpha > 4$ units, with increasing sideslip. This result was expected since the effects of coupling of the longitudinal and lateral directional forces and moments begin to dominate the equations of motion.

Conclusions

Results of the investigation of two methods for determining spin susceptibility using alpha-beta boundaries yielded the following conclusions:

1) Stability trends as a result of alpha-beta boundaries analysis using two different techniques were highly comparable. Differences were accountable to areas where equilibrium trimmed conditions were unobtainable.

2) Comparison of results obtained from static stability derivative analysis indicates that cross-coupling effects

dominate force and moment equations of motion at angles of attack greater than 2.5 units and angles of sideslip greater than 2 units; extended analysis techniques add further insight to results obtained from the classical $C_{n_{\beta_{dyn}}}$ analysis.

Appendix

The basic aircraft aerodynamic coefficients C_L,C_D, C_y, C_m, C_l, and C_n are shown as functions of angle of attack and sideslip in Figs. A1-A6. The aircraft as can be seen in Fig. A4 is statically unstable at low and intermediate angles of attack. In the analysis presented in this paper since lateral directional instability was of primary importance a perfect stability augmentation system was assumed to yield a 10% static margin for all flight conditions.

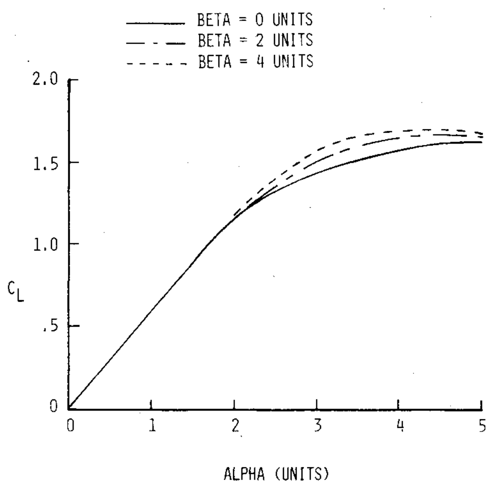

Fig. A1 Lift coefficients.

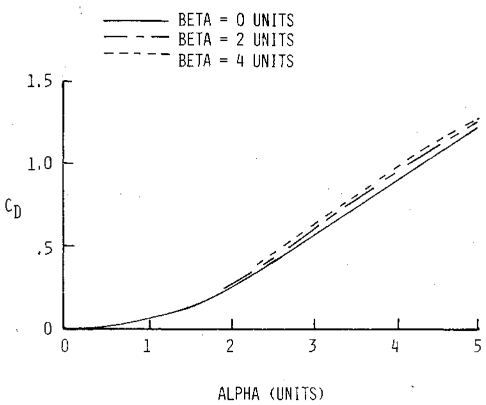

Fig. A2 Drag coefficient.

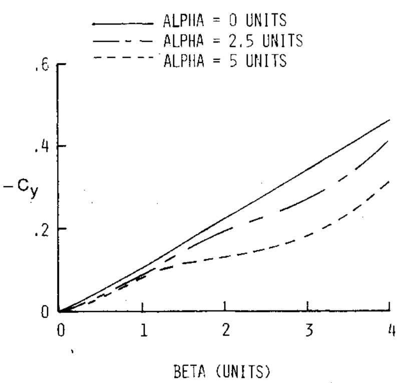

Fig. A3 Side force coefficient.

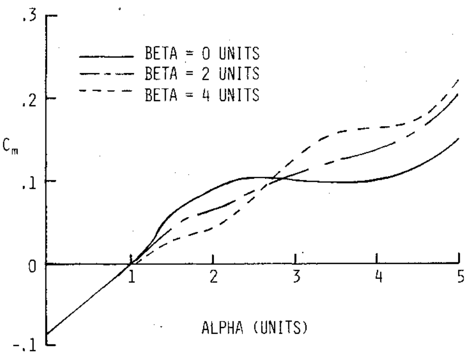

Fig. A4 Pitching moment coefficient.

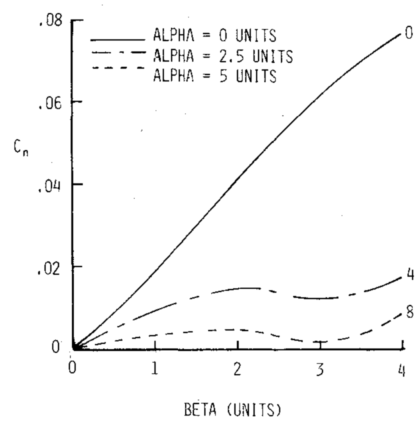

Fig. A5 Yawing moment coefficient.

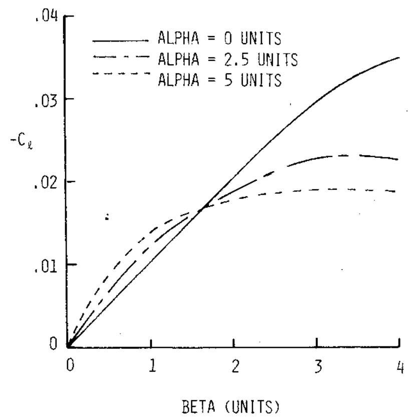

Fig. A6 Rolling moment coefficient.

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